Abraham-like return to constant c in general relativity: “$\mathfrak{R}$-theorem“ demonstrated in Schwarzschild metric

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Abstract

General relativity allows for a mathematically equivalent version in which length changes absorb the traditional changes in c. This conjecture is deomonstrated for the special case of the radial Schwarzschild metric. Two size-change results obtained a decade ago in the context of the equivalence principle – one relativistic, one quantum – are re-obtained in the radial Schwarzschild metric. Hence a previously neglected radial observable defined by $d\mathfrak{R}/dr = 1/(1–2m/r)$ determines physical distance. Since $d\mathfrak{R}/dt \equiv c$, Max Abraham’s constant-c postulate of 1912 is unexpectedly fulfilled. The well-known infinite “radar distance“ of the horizon of a Schwarzschild black hole therefore reflects an infinite distance. An infinite proper infalling time into black holes is a corollary. Since the latter time is canonically finite, an *anomaly* is encountered. To help decide it, an independent second proof is sketched based on a standing vertical light wave. An added merely qualitative third proof involves the Finkelstein diagram. If the new result can be confirmed, finished black-hole horizons, wormholes, Hawking radiation, charged black holes and singularities cease to exist in nature. Quantum-supported linear and curvature-supported nonlinear features of spacetime can be distinguished. ElNaschie’s fractal E-infinity theory offers itself as an independent test bed.
(April 9, 2007, March 20, 2008)‘

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1. Introduction

Einstein first introduced a height-dependent c (in a high tower on earth or equivalently an ignited long rocket in outer space) in the context of the equivalence principle in 1911 [1]. This proposal caused grave concern on the part of his elder colleague Abraham who, after having fully embraced Einstein’s special relativity, was reluctant to sacrifice the latter’s central tenet of a globally constant speed of light c [2]. Einstein’s new axiom of a potential-dependent c was instrumental to further progress and got eventually incorporated into general relativity four years later, as is well known [3].

The variable-c axiom has a familiar consequence in the Schwarzschild metric, which is the single most important solution of the Einstein equation of 1915. The “coordinate speed of light“ c(r) is here a function of the distance parameter r:

$$c(r) \;=\; \frac{dr}{dt} = c \cdot (1 - 2m/r) \;, \qquad (1)$$

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where 2m is the Schwarzschild radius (with $2m \equiv 2GM/c^2$, M the central mass, G Newton's gravitational constant and c the universal speed of light (cf. Foster and Nightingale [3], p. 129). Eq.(1) states that the speed of light valid with respect to the distance parameter r, c(r), becomes zero as r approaches the Schwarzschild radius 2m from above.

In spite of its well-known lack of constancy relative to r, c is bound to remain at least *locally* unchanged by virtue of Einstein's covariance postulate (which posits that locally, the laws of nature must be everywhere the same including the speed of light). That this constraint is indeed fulfilled by Eq.(1) can be seen as follows: Proper time τ is locally *slowed down* by the factor $(1–2m/r)^{1/2}$ relative to coordinate time t (since $d\tau = (1–2m/r)^{1/2}dt$; [3], p. 127). This is the same factor by which the radial distance R is locally *increased* relative to coordinate distance r (since $dR = (1–2m/r)^{-1/2}dr$; [3], p. 125). The two local changes – the temporal and the spatial one – taken together compensate for the change in c given by Eq.(1). Indeed $dR/d\tau = dR/dr \cdot dt/d\tau \cdot dr/dt = (1–2m/r)^{-1}\, dr/dt \equiv c$.

The *global* change in c formally implicit in Eq.(1) conflicts with Abraham's intuition. Could it be that, contrary to appearances, Abraham's postulate is actually *fulfilled* in the Schwarzschild metric, and if so in general relativity at large? The answer to this question is in the positive as far as the radial Schwarzschild metric is concerned. This surprise result is to be demonstrated in the following along with some implications.

2. The size-change conjecture

In 1998, an in principle well-known but rarely (if ever) mentioned relativistic fact was independently spotted in the equivalence principle: *inequality* of the two vertical radar distances (down-up and up-down, respectively) in an accelerating rocket [4]. The method used was the "WM-diagram." The two mirror-symmetric capital letters W and M stand for light rays moving updown or downup twice, respectively (forming a symmetric XXXX pattern). The diagram illustrates that *time intervals* along the top and the bottom of the 4 concatenated X's (that is, "upstairs" and "downstairs" in a vertically accelerating rocket) interlock consistently with each other *despite* their unequal durations. While this fact is well-known in principle (compare the "Einstein synchronization" of Rindler [5]), the pictorial method – which grew out of a chaos-theoretic mapping proposal made by Dieter Fröhlich – reveals a new fact: *relative size increase downstairs by the redshift factor observed from upstairs*. This is because the vertical distance, when measured using light pulses from upstairs, is exactly so much larger than when measured from downstairs. Conversely, the blueshift factor observed downstairs implies an equal *relative size decrease upstairs* by the blueshift factor observed downstairs, which amounts to the same thing. (The objection that *width* appears unchanged from the respective other vantage point can be met by invoking projective anisotropy.) The relative size change *explains* the unequal vertical radar distances found in the equivalence principle. The latter are, by the way, easy to verify empirically using a TV tower, a pocket laser, a mirror and a counter (Gerhard Schäfer, personal communication 2001). The *size change result* is, by the way, already implicit in a special-relativistic finding of Walter Greiner's [6].[1)]

In the same year 1998, Heinrich Kuypers came up with the idea to have a look, likewise in the equivalence principle, at the gravitational Dopplershift of *matter waves* in order to see how quantum mechanics fits in. This allowed him to realize that, if photon mass downstairs is reduced by the gravitational redshift factor as is well known [7], *any mass* on the same level must be reduced by the same factor owing to local energy conservation [8,9].[2)] Hence *quantum mechanics* predicts (via the de Broglie wave-length of matter waves and, more specifically, the Bohr radius which is inversely proportional to electron mass) that the *size* of every object downstairs is enlarged in proportion to its redshift [8,9]. This quantum prediction *coincides* with the previous relativistic prediction in a kind of pre-established harmony.

The two 1998 observations were each made independently of Abraham's conjecture. *A priori* it appears infinitely unlikely to suspect a connection. Or could it be that Einstein and Abraham are *reconciled* by Fröhlich and Kuypers? It is this outlandish conjecture which is to be demonstrated in the following. Since the "playground" of the equivalence principle is no longer sufficient, the Schwarzschild metric offers itself as the ballpark of choice.

3. Demonstration of the conjecture

3.1 Some well-known findings

The Schwarzschild metric is the oldest explicit solution of the Einstein equation. It was already found in late 1915 by a friend of Einstein's under unfavorable personal circumstances (Karl Schwarzschild died soon thereafter). The mentioned book by Foster and Nightingale [3] will continue to be used in the following as a backdrop – with page numbers put in brackets, like (p. 130), referring to their book.

Just as it was the case before with the equivalence principle [4], the up-down and the down-up distances measured by light sounding ("radar distances") differ by the mutual redshift (or, in the opposite direction, blueshift) factor *also* in the radial Schwarzschild metric. This fact deserves to be looked at in more detail.

Firstly, the mutual redshift factor owes its existence to the unequal *proper times* valid upstairs and downstairs. "Proper time" τ is, as already mentioned in the Introduction, at every local r defined by

$$d\tau = (1-2m/r)^{1/2}\, dt \tag{2}$$

if t is the coordinate time (p. 127).

Secondly, the "coordinate time difference" Δt between upstairs and downstairs depends on the *coordinate values* of the *o*uter (r_o) and *i*nner (r_i) radial position, on the one hand, and the local *coordinate speed of light c(r)* given by Eq.(1), on the other. Integration of Eq.(1) if written in the form $dt = c^{-1}(1-2m/r)^{-1}dr$, between r_i and r_o, yields the *coordinate time difference* valid for a down-up (or equivalently up-down) light signal:

$$\Delta t = \frac{1}{c}\int_{r_i}^{r_o} (1-2m/r)^{-1} dr \tag{3}$$

(p. 129). Multiplication of this time interval by c formally generates a corresponding *distance*:

$$c\Delta t = \int_{r_i}^{r_o} (1-2m/r)^{-1} dr \ . \tag{4}$$

This distance has *no name* up until now. (Only the indefinite version of the same integral is well-known under the name " *r** " in the Eddington-Finkelstein formalism [10], a fact that we shall come back to below.)

The distance given by Eq.(4) cannot be measured directly. It can only be evaluated on either end – where it is then automatically weighted by the local time-shrinking factor of Eq.(2). What comes out is the well-known "radar-sounding light distance" (as Foster and Nightingale call it [3], p. 130). The latter reads, when evaluated from the *upper* end r_o,

$$c\Delta t_o \;=\; d\tau_o/dt \cdot c\Delta t \;=\; (1-2m/r_o)^{1/2} \int_{r_i}^{r_o} (1-2m/r)^{-1}\,dr$$

or after integration

$$c\Delta t_o \;=\; (1-2m/r_o)^{1/2} \left(r_o - r_i + 2m \ln \frac{r_o - 2m}{r_i - 2m} \right) \qquad (5)$$

(p. 130). One sees that this *down-up radar distance* – as it can be called – diverges (becomes infinite) as r_i approaches the Schwarzschild radius 2m from above.

In corresponding fashion, the opposite radar distance $c\Delta t_i$ valid at the *lower* end r_i is arrived at. It differs from the former only by the subscript (i instead of o) in the first bracket:

$$c\Delta t_i \;=\; (1-2m/r_i)^{1/2} \left(r_o - r_i + 2m \ln \frac{r_o - 2m}{r_i - 2m} \right) . \qquad (6)$$

This *up-down radar distance* – as it can be called – unlike the former does *not* diverge when r_i (now the position of the observer) approaches the Schwarzschild radius 2m from above.

The *ratio* between the two different radar distances, Eq.(5) and Eq.(6), is

$$\frac{c\Delta t_o}{c\Delta t_i} \;=\; \left(\frac{1-2m/r_o}{1-2m/r_i} \right)^{1/2} . \qquad (7)$$

This ratio is the "WM result" of reference [4] valid in the Schwarzschild metric.

So much for some well-known facts in the radial Schwarzschild metric. Only the distinction made between "down-up" and "up-down" radar distance appears to be new.

3.2 Compatibility with the Fröhlich-Kuypers size change

The described facts of the Schwarzschild metric can now be *juxtaposed* with the surprise observation of Fröhlich and Kuypers – the redshift-proportional size-change principle – in order to see how well the latter fits in or whether it creates an incompatibility at some point which would then spell the end of the present approach.

The new point heuristically to absorb into the Schwarzschild metric is the redshift-proportional relative *size increase* downstairs predicted by Fröhlich and Kuypers in two independent contexts. Does this feature if hypothetically introduced *contradict* the accepted facts in the Schwarzschild metric? Surprisingly, the answer is *no*.

To see this, it is first necessary to realize that the Schwarzschild metric already *contains* a height-dependent change in size (which by the way *likewise* fails to show up in the transverse direction owing to projective anisotropy when looked at from above or below). This *canonical radial size increase* reads, as already mentioned in the Introduction,

$$\mathrm{dR} \;=\; (1\text{–}2\mathrm{m/r})^{-1/2}\mathrm{dr} \qquad (8)$$

(p. 125). After integration, this generates the so-called “radial distance“ between r_i and r_o:

$$\Delta\mathrm{R} \;=\; \int_{r_i}^{r_o}(1-2m/r)^{-1/2}\,dr$$

(p. 128), or explicitly

$$\Delta\mathrm{R} \;=\; [r_o(r_o-2m)]^{1/2}-[r_i(r_i-2m)]^{1/2}+2m\ln\frac{r_o^{\,1/2}+\left(r_o^{\,1/2}-2m\right)^{1/2}}{r_i^{\,1/2}+\left(r_i^{\,1/2}-2m\right)^{1/2}}\;. \qquad (9)$$

Note that this *traditional radial distance* does *not* diverge when r_i approaches the Schwarzschild radius 2m from above. Indeed, of the 4 radial distances identified so far in the Schwarzschild metric – Eqs.(4), (5), (6) and (9) –, only the first two diverge.

However, the “intrinsic local size change“ dR, valid in the Schwarzschild metric with respect to the local distance parameter r by virtue of Eq.(8), is not the end of the story in our present context since there now possibly exists a *new* local size change – the one predicted by the above-mentioned combined WM and de-Broglie argument. This postulated new size change is governed by the relative redshift or blueshift valid at the respective other radial position. Hence it is determined by the ratio of frequency shifts, Eq.(7), *divided* by the local proper-time factor valid at the observing position r_o by virtue of Eq.(2). This yields the predicted net factor

$$\left(\frac{1-2m/r_o}{1-2m/r_i}\right)^{1/2}\cdot\;(1\text{–}2\mathrm{m}/\mathrm{r_o})^{-1/2}\;\equiv\;(1\text{–}2\mathrm{m}/\mathrm{r_i})^{-1/2}$$

for any object located at r_i observed from $r_o>r_i$. Thus, we have (writing r for r_i in the brackett)

$$\mathrm{d}\rho \;=\; (1\text{–}2\mathrm{m/r})^{-1/2}\,\mathrm{dr} \qquad (10)$$

as our conjectured new local size change factor.

The *postulated* new local size-change dρ of Eq.(10) has exactly the *same* form as the local size-change dR of Eq.(8) above. Therefore there are *two* possibilities open at this point: Either the new size change factor of Eq,(10) is nothing but a new re-derivation of the old factor of Eq.(8); then the traditional radial distance R of Eq.(9) remains the only physically relevant radial distance in the Schwarzschild metric. Or *both* size change factors (the old dR/dr and the new dρ/dr) contribute on an equal footing locally if the new size change of Fröhlich and Kuypers is real. In this case the resulting “effective local size change factor“ d$\mathfrak{R}$/dr is equal the product of the two individual factors named:

$$\frac{d\Re}{dr} = \left|\frac{dR}{dr}\cdot\frac{d\rho}{dr}\right| = (1-2m/r)^{-1} ,$$

that is,

$$d\Re \;=\; (1\text{–}2m/r)^{-1}\,dr\,. \tag{11}$$

This hypothetical new effective local size change factor generates a *new distance integral*:

$$\Delta\Re \;=\; \int_{r_i}^{r_o}(1-2m/r)^{-1}dr \;=\; \left(r_o - r_i + 2m\ln\frac{r_o-2m}{r_i-2m}\right)\,. \tag{12}$$

The new distance integral $\Delta\Re$ ("$\Re$-distance") *replaces* the traditional distance integral ΔR of Eq.(9) as the correct "radial distance" – *if* the new Fröhlich-Kuypers size change factor is added while everything else remains unchanged.

Unexpectedly, Eq.(12) is *identical* to Eq.(4) above. Thus *nothing* has been introduced in effect as far as measured distances are concerned! The above employed "roundabout way" of heuristically using *two* local size changes – the old Schwarzschild factor of Eq.(8) and the hypothetical new Fröhlich-Kuypers factor of Eq.(10) – in order to explain the old radar distance of Eq.(4) proves to be a perfectly legitimate option. This option renders the traditional position-dependent *reduction of c* of Eq.(1), which likewise leads to Eq.(4) ($\equiv$ Eq.12), redundant. Both views make equal sense at first sight. So one should let nature have a word. The new view if false should lead to predictions at variance with reality. Is this the case?

3.3 The Shapiro time delay

The Shapiro time delay was introduced in 1964 by Shapiro [11] and independently by Muhleman and Richley [12] as a testable counterintuitive implication of the Schwarzschild metric ("fourth test of general relativity"). They encountered much skepticism at first. To date, the underlying equation (Eq.3) is empirically confirmed in the solar system to an accuracy of $2\cdot10^{-5}$ [13]. The currently accepted interpretation is that time suffers a counterintuitive delay while the radial distance R is covered and that this delay is predictably caused by the slowing of the velocity of light c(r) near a gravitating object.

But there now exists an alternative interpretation: the new size change axiom of Eq.(11) can be invoked. Adopting this interpretation is equivalent to saying that it is "not a change in c but a change in distance" that has been measured. This means that the two identical distances, cΔt of Eq.(4) and $\Delta\Re$ of Eq.(12), can both be re-named into a *single* distance,

$$R_A \;=\; \left(r_o - r_i + 2m\ln\frac{r_o-2m}{r_i-2m}\right)\,. \tag{13}$$

3.4 Abraham vindicated

The newly obtained unique distance R_A produces (after division by c) the very time delay Δt familiar from Eq.(3) above (with ensuing radar distances Eqs.5 and 6). The old local size change factor of Eq.(8) valid in the Schwarzschild metric ceases to be alone since a new factor, Eq.(10), has been brought in.

That *both* factors are valid in the Schwarzschild metric (in the product of Eq.11) comes as a surprise from the point of view of the equivalence principle. Here it is not the new factor of Eq.(10) which is surprising but the fact that it no longer stands alone in determining size in the Schwarzschild metric due to the presence there of the *old* factor of Eq.(8). This amounts to a *qualitative difference* between the "curved" Schwarzschild metric and the "flat" equivalence principle. Quantum mechanics continues to "see" only the flat version and so do mass and energy. Only *size* (and with it distance) is determined by *both* factors.

If we accept the new size change law (Eq.11) as being valid in the Schwarzschild metric: what about Abraham's hunch? The new-old distance found (Eq.13) deserves to be given a new name: "Abraham distance" – R_A. Why? Because this distance (Eq.12≡Eq.4) formally implies that *c is constant* over the whole trip! This fact was already implicit in Eq.(4) above – but our eyes were held at the time as it were since we did not yet have a good reason to take the coordinate-time difference Δt of Eq.(3) *that* seriously.

The new "Abrahamian interpretation" of Eq.(13) is *equivalent* to the standard interpretation of the radial Schwarzschild metric – as far as predicted redshifts, time delays for light and any resulting formal distances are concerned – yet with *c globally constant*. Hence we can state the following "$\mathfrak{R}$ theorem":

Theorem: *In the radial Schwarzschild metric, global constancy of c holds true with respect to the* natural *distance parameter $\mathfrak{R}$, defined by $d\mathfrak{R} = (1–2m/r)^{-1}dr$.*

The *naturalness* follows from the Fröhlich-Kuypers size-change. The *validity* follows (using Eqs.11 and 1) from the identity $d\mathfrak{R}/dt = (1–2m/r)^{-1}dr/[dr(1–2m/r)^{-1}/c^{-1}] \equiv c$.[3)]

A more general way to put the same result would be to speak of the "conservation of longitudinal spacetime volume" (longitudinal spacetime area) in the radial Schwarzschild metric – and presumably general relativity at large. In the present context, the formulation that "Abraham's dream" is fulfilled for once in general relativity in the special case of the radial Schwarzschild metric, is perhaps the most appropriate.

4. Consequences

4.1 First, the familiar side

The unified picture arrived at does not change anything in the accepted facts. Only on the level of *interpretation* are there any consequences to expect. One such interpretational consequence is, nevertheless, quite tangible:

Corollary: *The horizon of a Schwarzschild black hole has an infinite distance R_A (≡$\mathfrak{R}$) from the outside.*

This infinite-distance result does not really come as a surprise because the "radar distance" (signal-return time multiplied by ½c) of the horizon is well-known to be infinite from above by virtue of Eq.(5) as we saw ([3], p. 130). In the present context, this familiar finding acquires a subtle change of meaning, however: The infinite radar distance is no longer an “artifact“ of the *change-in-c* downstairs, as had to be assumed up until now, but the consequence of a previously overlooked *change-in-size* downstairs. According to the achieved new semantics, the same distance thus is *really* infinite from above. This conclusion is in perfect agreement with the Abraham principle. Everything appears harmonized for once.

4.2 A surprise secondary implication

In spite of the harmony obtained, there exists a derived *secondary* implication which appears virtually unacceptable: Black holes can now no longer be reached in finite time – not only by *light* with its infinite radar-sounding delay for which this fact is well known as we saw (Eq.5 and Shapiro), but by *any* infalling object. The result is so strong it even remains true when the falling time is measured in terms of the *proper* time of the infalling object itself! For the relative distance is now “really infinite“ (R_A is infinite for $r_i \equiv 2m$). Hence the above “change in semantics“ is *more* than a mere change of words for once: it has tangible physical consequences. Since this cannot be the case by very definition, some previously accepted *physical facts* are bound to have been in error!

This statement amounts to an *anomalous situation* having been reached. Hence the anomalous “infinite proper infalling time“ merits an independent proof in terms of the *standard picture* since the physics is bound to be invariant under a change of semantics. If such a proof were to be found, the accepted ways of deriving the contrary – dating back to Oppenheimer and Snyder’s famous paper of 1939, cf. [14] – would lose credit. The at first sight more natural thing to do – to re-work the old equations themselves – would be counterproductive, given that the pertinent mathematical paths have all stood the test of time. Only a round-about way – like the cat‘s around the hot mush – has any chance to succeed in case there *really* is something out of kilter. Such an alternative proof can tentatively be based on the paradigm of a *standing light wave* (generated by two mutually opposite laser sources of perfectly matching frequency and phase, cf. [15]). A rough sketch goes as follows:

A standing light wave is assumed to be set up vertically between the horizon and the outside world. This can be achieved in principle: two mutually opposite laser canons of *differing* frequencies can be positioned upstairs and downstairs in such a way as to generate a standing light wave in between them – if the frequency ratio matches the mutual redshift or blueshift factor (Eq.7). (If necessary, mediating “doubly open laser canons“ tuned to the locally matching intermediary frequency can be inserted.) In the extremal case – outside-to-horizon – at stake, the frequency ratio between downstairs and upstairs approaches infinity. In this limit, the resulting “Jacobian ladder of light“ possesses an *infinite* number of rungs (standing wave-crests). This prediction is in accord with the accepted infinite radar distance (Eq.5). While the locally valid distance between rungs differs widely – approaching zero for people living near the horizon –, the distance between rungs is *constant* for a fictitious particle falling at constant speed. Note that according to the equivalence principle, a *light wave* sent down from above *retains* its frequency in the upper frame in spite of its being progressively shortened when arriving at – or passing by – a more downstairs position. The same features-conserving fact holds true for a constant-speed particle that is slower than a photon. However, the speed of a falling ordinary particle is *not* constant but accelerating by definition. The situation is exactly the same as it holds true for any other ladder of infinitely many equi-spaced rungs – in special relativity. In special relativity, an infinite number of equi-spaced wave crests *cannot* be passed by in finite proper time – neither at constant speed nor at constant acceleration nor (as here) under an increasing but flattening-out acceleration; compare Eq.(5.24) of French [16] with the pertinent classical exercise (20.2) of Greiner‘s book ([6], p. 168). This result carries over via the equivalence principle. Hence the total proper infalling time is *infinite*. (Q.e.d.) 4)

The result just sketched is in accord with the infinite distance of Eq.(13) above. Still, since the time-honored reigning consensus holds that the Schwarzschild metric implies a *finite* proper infalling time (cf. [3], p. 139, or [14], p. 851), a third, only *qualitative*, argument appears desirable to have as well:

Pictures come to mind at this point. More specifically, the fact that the "coordinate speed" of an infalling body, v(r) = dr/dt ([3], p. 143), needs to be constantly *adjusted* to the local "coordinate speed of light" c(r) = dr/dt of Eq.(1). This "consistency check" is particularly vital at coordinate values close to the horizon where the radial light cones become more and more compressed around the curves of infalling matter near the asymptotic vertical line r = 2m of the horizon, in the traditional r,t diagrams. While a detailed account of the local situation is not possible in such drawings, there is one exception: the Finkelstein diagram ([17], p. 152). Here, the ingoing light rays are straight 45-degree ascending lines that, nevertheless, are subject to a (graphically invisible) *exponential scaling* in the neighborhood of the vertical line r = 2m. The same applies to the almost parallel slightly less slanted particle rays. Since in this diagram, r*+t is plotted versus the horizontal r axis ([17], p. 150), the Finkelstein diagram is compatible with Eq.(13) above. For $r^* = R_A$ in this diagram (as mentioned above following Eq.4). Although this pictorial argument is only *qualitative*, as ordered, it can possibly even be made quantitative (q.e.d.).

4.3 Consequences of the new unreachability

Firstly: If the horizon cannot be *reached* in finite time by any object, black holes also can no longer even *form* in finite time. For a horizon that cannot be reached in finite time can also not arise in finite time. (What precisely happens when just the "last iota" of mass remains to be added to an almost-critical homogeneous collection of masses, represents an interesting selforganization-type question; note that action-at-a-distance cannot be invoked in this context.) From the nonexistence of a finished horizon it then follows that Hawking's beautiful evaporation result [18], which relies on a finished horizon, gets infinitely delayed, too, and hence ceases to be physically effective. This rule remains valid for mini-black holes (despite their greater tunneling capabilities) by virtue of Kuypers's quantum-scaling result.

Secondly: Light cones cease to be compressible in the radial direction of the Schwarzschild metric. This fact is bound to have further consequences – in the context of time machines and other very general implications of the Einstein equation (like gravitational waves and rotating frames). For example, wormhole-based time machines [19] depend on the horizon being reachable in finite time. They therefore are automatically ruled out in the Abraham picture. Gödel time machines, on the other hand, remain possible (compare the beautiful drawing in [17], p. 169). This fact notwithstanding, a cautioning remark recently offered by a youngster should perhaps not go unmentioned ("Time machines cannot exist given the infinite duration of the future." *Why?* "They would be all over the place by now." *Unless the percentage of time travellers that aren't infinitely careful about camouflage is zero.* "Yes – but this is unthinkable!").

4.4 Main open task

The revived Abraham proposal of a universal c amounts to a surprise implication of the radial Schwarzschild metric. Is it possible that alternative metrics derived from the Einstein equation will teach otherwise? The mentioned qualitative fit with the Eddington-Finkelstein metric speaks in favor of reconciliation. Therefore, the next open task to solve reads: How do the field equations *themselves* look like if “size, not c“ depends on the gravitational potential?

5. Discussion

A simple new result valid in a subcase of the Schwarzschild metric was presented. “Radial spacetime-volume conservation“ is one possible way to put it. The slower the time locally, the larger space locally. The stronger the magnification of time, the stronger the magnification of space: hence “space-over-time“ is constant – c. Max Abraham would have liked this result. A first glimpse of how his mind worked I got from Valérie and Christophe Letellier at the university of Rouen three years ago.

The result presented is nothing but a beginning. Nonradial directions in the Schwarzschild metric have yet to be considered. Angular momentum has to be introduced next (Kerr metric). And the full Einstein equation is waiting to be considered thereafter. Even more sophisticated higher-dimensional analogous equations [20,21] are bound to come next.

What will remain if the main result can be confirmed? Four results are likely to persist:
1) Nonexistence of finished horizons (due to an infinite proper infalling time) and hence nonexistence of finished black holes, so that only “almost black holes“ [22] remain.
2) Nonexistence of Hawking radiation.
3) Nonexistence of any spacetime elements beyond the horizon (including singularities).
4) Nonexistence of charged almost black holes.

These four predictions are surprising because they each fly in he face of accepted wisdom. If they hold true in the radial Schwarzschild metric, analogous new results are bound to be found in the four less restricted cases mentioned. It hence would be nice to have a simple method to falsify the above result. An independent approach to quantum spacetime was found by ElNaschie [23], cf. [24]. It will be instructive to see whether part or all of the above predictions can be confirmed or disproved in this independent methodology.

To conclude, a so-called “variantological approach“ to spacetime physics has been presented. That is to say, a fictitious return to an earlier level of sophistication was heuristically adopted [25]. Whether the presented approach can stand the test of time is open. Possibly – or hopefully –, it can be falsified soon since its results challenge too many accepted facts in the modern fabric of spacetime. A priori speaking, the probability that the two simple insights of Fröhlich and Kuypers can turn back the wheel of history to a time when Einstein and Abraham fought their friendly battle of giants is negligibly small. Where precisely is the error located?

Acknowledgments

I thank Dieter Fröhlich and Heinrich Kuypers for their fundamental contributions and Jürgen Parisi, Frank Kuske, Martin Pfaff, Valentin Letellier, Walter Greiner, Ken Ford, Theodor Hänsch, René Thomas, Michael Baune, Michael Maschke, Bill Seaman, Jonathan Kemp, Martin Howse, Martin Lübcke and Peter Weibel for discussions and Jörg Frauendiener for vital criticism. Michelangelo Mangano provided late-hour encouragement. For J.O.R.

Footnotes

[1)] The size change can also be derived from the twin-clocks paradox of special relativity: Conservation of angular momentum implies that the "younger clock" (if implemented as a frictionless rotator) must have been proportionally *enlarged* while making its fewer turns. Cf. [8,9] for an analogous implication of the gravitational twin paradox.

[2)] The same fact was mentioned in passing by Werner Israel [26]: Quote: "the gravitational (..) redshift factor (..) recalibrates locally measured mass and work to energies available to an observer at infinity."

[3)] Note, by the way, the interesting identity $d\mathfrak{R}/dt \equiv dR/d\tau$ (see Introduction).

[4)] But see Added in proof.

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Added in proof: reception, erratum, confirmation

Reception. With several thousand downloads, the above paper is one of the most-read in general relativity. An early all-out criticism by the prestigious Einstein Institute [27] got subsequently replaced by a toned-down version in late July [28]. The latter no longer repeated the claims contained in the first that the Abraham paper contradicted both general relativity and experiment. The only relevant criticism that remained was a prediction: that *if* the gothic-R theorem were to be extended from the radial Schwarzschild metric to the full metric, it would prove incompatible with celestial mechanics. Even this conditional prediction had already been laid to rest by the successful re-formulation of the full Schwarzschild metric in terms of the gothic-R variable, achieved by an anonymous author signing with "Ich" [29] (see his Eq.17). No further claim at falsification has been made to my knowledge. All the high-publicity claims at falsification made since by high-ranking institutions (like KET, CERN and two national parliamentary bodies) rely on the authority of Nicolai's first paper and are, therefore, baseless as far as I can see. So are the ill-fated experimental decisions made in their wake.

Erratum. The merits of the already mentioned anonymous author do not stop here. He also succeeded in finding a first error in the above paper – not in the gothic-R theorem proper but in the added conjecture (erroneously labeled "Q.e.d.") that the proper infalling time were infinite. This conjecture is false: the proper infalling time is *finite* [30]. The reasoning is based on the Rindler metric which is a valid approximation to the Schwarzschild metric [5,31]. The new result at first sight comes as a surprise from the point of view of the gothic-R theorem since the infinite distance implicit in the latter cannot be covered in finite proper time by definition – unless luminal observer speeds are involved. This appears absurd at first sight since observers are massive bodies and massive bodies cannot reach luminal speeds in finite time. Surprisingly, Ich's result goes hand in hand with a corollary that implies exactly this.

Confirmation. The new confirmative corollary follows from the Rindler metric. Since the Rindler metric involves only special relativity, it can be fully understood in terms of a 2-D Minkowski diagram (the familiar x,t frame of special relativity). The Rindler metric – if I may dwell on it a bit more – refers to a long rocket in constant acceleration in outer space, with earth's gravity (1 g) reproduced at the tip. The full rocket consists of many segments each carrying its own pair of boosters on the outside. (Picture many solid hollow cylinders pairwise connected by a rubber tube.) In the x,t plane, the trajectories of all segments come to lie in between $t = + x$ (right-hand part of first bisector) and $t = - x$ (second bisector). This *quarter* of the full Minkowski plane is called the "Rindler wedge." Inside the wedge, we have our 1-light-year-long rocket, momentarily located motionless along the x-axis while accelerating all along at at full blast while stretching from $x = 0$ (bottom) to $x = 1$ (tip). In the x,t plane, the trajectory of the tip then rises up vertically to gently bear right along a curved line in the form of a half-hyperbola that asymptotically approaches the first bisector to asymptotically reach it at $t = x = \infty$. The lower (past) part of the same trajectory does the same thing reflected downwards, approaching the second bisector in negative infinite time. (This means, physically speaking, that the constant acceleration is superimposed onto a constant negative, initially at $t = -\infty$ luminal, speed.) The more inner segments of the rocket ($x < 1$ on the x-axis) all do the same thing along proportionally downscaled full hyperbolas having a correspondingly larger constant acceleration each (g/x locally). This principle continues right down to the 90-degree angle at $x = 0$ (the origin) where the acceleration becomes infinite (g/0). The assumed gradient in accelerations is necessary in order for the rocket to remain connected over time – an accepted if paradoxical fact in special relativity [5]. It follows that the intra-rocket times ("rocket tip times") T remain definable indefinitely – all along straight lines through the origin. The bundle of these "T times" ranges, from $T = -\infty$ at slope – 1, via $T = 0$ at slope zero, to $T = +\infty$ at slope = + 1 [5]. All T-times are on the same footing, that is, can each be identified with the x-axis on shifting the initial condition by simply "scrolling up" or "scrolling down," respectively.

Now the two results announced. First, the *finite* proper infalling time result [30]: The internal observer at the tip of the rocket (at $x = 1$ and $t = 0$) lets go of his handle and simply stays put while moving up in time t along the $x = 1$ vertical. He then simultaneously is "falling" freely inside the rocket – so as to leave it through an opening in the bottom at $t = 1$ (1 year) at the point $x = t = 1$ while the rocket's bottom departs from him at the speed of light. He at this point has effectively "fallen" through the whole length of the rocket in 1 year of his proper time.

Second, the new *luminal-speeds* result: To best see it, we assume for starters that the hole in the tail had been plugged by a trampoline (the asssumption can be dropped later). The coasting passenger – if resilient enough – then bounces back all the way up toward the tip in another year of his proper time. In the Rindler diagram, this rebouncing trajectory is again a straight line: starting at the point $x = t = 1$, it continues along the first bisector in coincidence with the latter so as to let the jumper re-catch his handle, which contimued along the curved hyperbola of the rocket's tip, at $x = t = \infty$.

That is all. One sees that the two straight legs of the observer's trip are mutually equivalent (except for orientation in time). For it is possible to "scroll down" the initial time T when the observer lets go of his handle, all the way down from $T = 0$ (asumed so far) to $T = -\infty$. In the new equivalent picture, the observer reaches the trampoline, not at $x = t = 1$ but rather at x

= t = 0 (origin). This *symmetric* picture reveals that during either half trip (the two being mirror images of each other), an infinite distance in outer space is covered by the observer – in finite proper time! Hence there *always* exists an appropriately chosen frame in which an infinite distance is being bridged by the falling (or rebouncing) observer in finite proper time.

This new result is surprising since luminal speeds of massive bodies had no place in physics up until now. The reason they are a reality lies in the free choice of frames that is the hallmark of the Rindler metric, the above "scrolling operation." For we can always make sure that the "arriving event" at the bottom of the rocket (which is the horizon [31]) coincides with the origin of the metric (the 0,0 foot point of the Rindler wedge). This point can be reached from inside the wedge (or be left into the wedge, respectively) only along one of the two 45-degree trajectories, that is, along luminal trajectories coincident with one of the wedge's boundaries.

This fact – that the origin of the Rindler metric is "nonsingular" – comes as a surprise. Recall that the bottom of the rocket was factually reached by our first "falling" observer on his stepping out into the light from the hole in the rocket's bottom, at x = t = 1. This fact [30] now *also* means that a luminal speed is accessible to a falling observer or particle inside the Rindler metric. But cannot such a speed only be reached after an infinite period of constant acceleration by definition? This is correct. Amazingly, both seemingly contradictory facts are mutually compatible for once. For the waiting time under permanent constant acceleration inside the rocket, is infinite: The handle (or a companion sitting on the neighboring seat) has to wait upstairs an infinite period of time under constant acceleration, bridging an infinite distance in outer space in the process, before being at last reunited with the back-bouncing, youthful, observer.

More abstractly speaking, the "scrolling operation" *includes* the two 45-degree singular limiting cases – with their luminal speeds – as effective nonsingular cases. This mathematical finding is amenable to a deeper (differential-topological) explanation. Here, it suffices to note that such a situation – that the singular limits are nonsingular – is unheard of in physics. This fact gives the Rindler metric and its close relative, the Schwarzschild metric, a unique place in nature.

What does the effective infinite intra-rocket (and extra-rocket) distance found mean? It means that a well-known result possesses a new corollary. So far, it was known that *photonic* Hawking radiation by definition takes *zero* proper time to emerge from the horizon after having bridged the whole (infinite) gothic-R distance and now, *material* Hawking radiation analogously takes a *finite* proper time to come out. Similarly *light* takes an *infinite* external time to come out as we saw in the paper and now, *material* particles take a longer (twice as long) *infinite* outside time to come out across the whole gothic-R distance. Again, we only have reproduced a self-evident fact one feels.

Nevertheless the (now trivial) prediction of an infinite "emerging time" because of an infinite distance to be covered from the horizon, is what gave the above paper its worldwide attention. For this prediction implies that microscopic black holes generated on earth cannot evaporate in finite time – and hence put the planet at risk if earth-bound. In this way, an *ethical* dimension got suddenly attached to a pure-physics result. This dimension was, interestingly, not seen at the time of writing the paper but was the merit of a relativist colleague who when recommending publication jokingly wondered whether there could not be repercussions on the "LHC" experiment. Although I had never heard of the latter, the remark eventually triggered a vain attempt at defusing the joke. When it failed, a more serious attempt followed so it almost became a sport to hunt for a more sophisticated argument in order to defuse the joke. Each floundered for a different reason so that a vague hunch of a danger-conserving principle being at work formed – that all the uncanny failures may be non-coincidental. The suspicion turned tangible when the final unsuccessful attempt at giving the all clear had been communicated to CERN in May and published in July [32]: neutron stars seem to possess a special quantum protection against natural, cosmic ray-borne, very fast analogs to any miniblack holes potentially created on earth (superfluidity was the likely culprit). Eventually the idea of a joke played by nature on humankind – that the artificial slowness of human-made analogs could be a curse – befell the whole planet on September 10 when more than 500 newspapers across the globe referred to it in one way or the other. The joke still waits to be defused. Thinking twice (by no longer opposing the safety conference publicly demanded on April 18 [33]) remains an option to date following the felicitous *fehlleistung* that occurred at CERN on September 20. The whole globe is grateful for the second chance at falsification granted to it. Letting an idea die is always the less costly option according to Karl Popper.

I thank Gerhard Huisken for leading me to Robert Wald's book, Georg Slotta for references [29,30] and many exchanges that opened my eyes to the Rindler metric, Dieter Fröhlich, Christophe Letellier and Peter Kloeden for discussions and Andy Hilgartner, Artur Schmidt, and Kensei Hiwaki for stimulation. For J.O.R. 12/31/08.